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# GEOMETRY.

Conducted by B.F.FINKEL, Kidder, Missouri. All Contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

37. Proposed by B. F. BURLERSON, Oneida Castle, New York.

Inscribe in a semi-circle (1), a rectangle having a given area; (2), a rectangle having the maximum area.

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, and S. N. COLLIER, University of Mississippi.

(1). Let  $ABC$  be the given semi-circle with center  $O$ ; and let  $S$  be a square of given area with side  $a$ .

To inscribe in  $ABC$  a rectangle equivalent to  $S$ .

*Construction*:—At  $A$ , the left-hand extremity of the semi-circumference, draw a tangent  $AD$  making it equal to  $a$ . Draw  $OD$ . With  $O$  as center and  $OD$  as radius describe an arc cutting  $OA$  produced at  $E$ . Through  $E$  draw a line making an angle of  $45^\circ$  with  $EO$ , intersecting the circumference at  $B$ . Through  $B$  draw a parallel to  $AO$  cutting the circumference at  $G$ . Through  $B$  and  $G$  draw perpendiculars to  $AO$  meeting the bounding diameter at  $F$  and  $H$  respectively. Then  $FBGH$  is the rectangle required.

*Proof*:— $OD = \sqrt{AD^2 + OR^2} = \sqrt{a^2 + R^2}$ , denoting  $OA$  by  $R$ . But  $OD = OE = OF + FE = OF + FB$ .  $\therefore \sqrt{a^2 + R^2} = OF + FB$ .

Squaring,  $a^2 + R^2 = OF^2 + 2 \cdot OF \cdot FB + FB^2$ . But  $OF^2 + FB^2 = R^2$ .  $\therefore a^2 = 2 \cdot OF \cdot FB$ .  $2 \cdot OF$  is the base of the rectangle and  $FB$  is its altitude. Also,  $a^2 = \text{given area}$ .  $\therefore$  the rectangle is equivalent to the given area.

(2). Let  $ABC$  be a given semi-circle with center  $O$ . To inscribe in  $ABC$  a maximum rectangle.

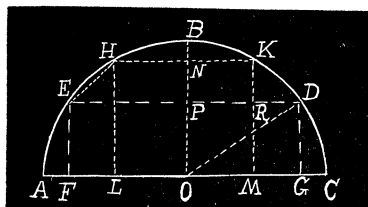
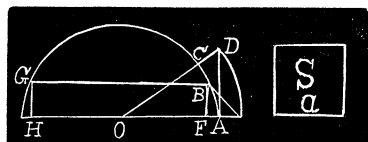
*Construction*:—Draw the radius  $OD$  making an angle of  $45^\circ$  with  $OC$ . With  $D$  as one vertex construct the rectangle  $DEFG$ .  $DEFG$  is the rectangle required.

*Proof*:—Let  $LMKH$  be any other inscribed rectangle;  $OB$  the radius perpendicular to  $CC$ .

Compare rectangles  $PG$  and  $NM$ , the halves of the rectangles  $EG$  and  $HM$ . Rectangle  $PM$  is common to the two. Rect.  $RG > \text{rect. } NR$ ; for  $DG (= DP) > RP$ , and  $DR > KR$ .

Since  $\angle RKD > \angle RDK$ , the former being measured by one-half of an arc greater than  $90^\circ$  and the latter by one-half an arc less than  $90^\circ$ .

It follows that  $EDGF$  is the rectangle required.



A. L. Foote furnished a neat algebraic solution; G. B. M. Zerr, P. S. Berg, Cooper D. Schmitt solved the problem by the calculus, and C. D. M. Showalter gave a good geometrical solution. Space forbids further consideration of this problem.

38. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Give a *strictly geometric* proof of my fundamental theorem of the Inscription of Regular Polygons, viz: Suppose a circle of unit radius divided at the points  $A, A_1, A_2, A_3, \dots, A_p, \dots$  into  $2p+1$  equal parts and the diameter  $AO$  drawn. Then, if the chords  $OA_1, OA_2, \dots, OA_p$  be drawn, we have  $OA_1 - OA_2 + OA_3 - OA_4 + OA_5 - \dots \pm OA_p = 1$ .

Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

For plainness we will solve this problem in full for the 13-gon. A general solution is as easy, but not as clearly understood.

Let  $OA_6, OA_5, OA_4$ , &c.  $= a_1, a_2, a_3$ , &c.

$$A_4 A_5 = A_5 A_6 = A_6 A_7 = \&c = c, A_4 A_6 = A_5 A_7 = A_6 A_8 = \&c = d.$$

Now by Ptolemy's Theorem:—The rectangle contained by the diagonals of a quadrilateral inscribed in a circle &c., we easily get the following relations:

$$\begin{array}{l|l|l} c(a_1 + a_3) = da_2 & c(a_6 + a_8) = da_7 & c(a_{10} + a_{12}) = da_{11} \\ c(a_2 + a_4) = da_3 & c(a_7 + a_9) = da_8 & c(a_{11} + a_{13}) = da_{12} \\ c(a_3 + a_5) = da_4 & c(a_8 + a_{10}) = da_9 & c(a_{12} - a_1) = da_{13} \\ c(a_4 + a_6) = da_5 & c(a_9 + a_{11}) = da_{10} & c(a_2 - a_{13}) = da_1 \\ c(a_5 + a_7) = da_6 & & \end{array}$$

$$\begin{aligned} & \text{Hence } d \{ (a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13}) - (a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12}) \} \\ & = c \{ (a_2 - a_{13}) + (a_2 + a_4) + (a_4 + a_6) + (a_6 + a_8) + (a_8 + a_{10}) + (a_{10} + a_{12}) + (a_{12} - a_1) \\ & \quad - (a_1 + a_3) - (a_3 + a_5) - (a_5 + a_7) - (a_7 + a_9) - (a_9 + a_{11}) - (a_{11} + a_{13}) \} \\ & \therefore (d + 2c) \{ (a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12}) \\ & \quad - (a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13}) \} = 0. \end{aligned}$$

$$\begin{aligned} & \therefore a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12} \\ & = a_1 + a_3 + a_5 + a_7 + a_9 + a_{11} + a_{13}. \end{aligned}$$

$$\begin{aligned} & \text{Generally } a_2 + a_4 + a_6 + \dots + a_{2p} \\ & = a_1 + a_3 + a_5 + \dots + a_{2p+1}. \end{aligned}$$

In the above,  $O$  can be any point between  $OA_p$  and  $OA_{p+1}$ .

In the problem  $OA = a_7 = 2$ ,  $a_6 = a_8$ ,  $a_4 = a_6$ ,  $a_2 = a_4$ ,  $a_1 = a_{13}$ ,  $a_3 = a_{11}$ ,  $a_5 = a_9$ .

$$\begin{aligned} & \therefore a_2 + a_4 + a_6 = a_1 + a_3 + a_5 + 1, \text{ but} \\ & a_6 = OA_1, a_5 = OA_2, a_4 = OA_3, a_3 = OA_4, \\ & \quad a_2 = OA_5, a_1 = OA_6. \\ & \therefore OA_1 - OA_2 + OA_3 - OA_4 + OA_5 \\ & \quad - OA_6 = 1, p, \text{ even.} \end{aligned}$$

For 15-gon.  $OA_1 - OA_2 + OA_3 - OA_4 + OA_5 - OA_6 + OA_7 = 1$ ,  $p$ , odd.  
 $\therefore OA_1 - OA_2 + OA_3 - OA_4 + OA_5 - \dots \pm OA_p = 1$ , as  $p$  is odd or even.

39. Proposed by J. K. ELLWOOD, Principal of Colfax Schools, Pittsburg, Pennsylvania.

If on the three sides of any plane triangle equilateral triangles be described, the lines joining the centres of these equilateral triangles form an equilateral triangle.

